Versatile FFT Supports Accurate 1/3 Octave Analysis

Technical Note TN-257

Introduction

When Agilent chose to retire the classic *HP 3582A audio spectrum analyzer*, high-quality audio spectrum analysis suddenly became a lot harder. This article discusses a way to configure a general-purpose xDAP measurement system for this purpose — discovering higher levels of performance, lower cost, and new opportunities for measurement system automation.

Overview of Octave Analysis

Mathematically, a pair of sine waves at 80 and 80+10 cycles per second, or at 8000 and 8000+10 cycles per second, are equally distinguishable. As audio signals, however, the first pair would be perceived as significantly different tones to a listener, while the last pair would be virtually indistinguishable. Pitch is perceived as changing with the ratio of frequencies, not by linear increase in cycles per second. In an audio signal, tones one octave apart differ by a factor of 2 in frequency. As you go from lower to higher frequencies, each successive octave band doubles in width.

To more naturally group frequencies of audio signals, so that the distributed signal power scales better for analysis, measured signal power can be combined within each octave. This is known as octave analysis. For purposes of analyzing audibility rather than equipment performance, A-weight filtering [2] is often combined with the octave analysis, so that instead of showing the actual signal power, each octave shows approximately the *perceived* signal loudness in each octave.

The audible range is spanned by just a few octaves, so partitioning the spectrum octave-by-octave produces a relatively coarse distribution. Resolution is improved by breaking the octave bands into sub-octave bands, preserving the logarithmic band spacing. For the conventional case that octaves are separated into 3 parts, each successive 1/3 octave band increases in width by a factor of `1.25992`. Though the relative widths are fixed mathematically, standards [3] define nominal locations for these bands — useful for labeling purposes.

Methods of Octave Analysis

The traditional way for instruments to perform an octave analysis was to pass the signal through a bank of analog bandpass filters, each filter responding to a narrow portion of the spectrum. The total signal power in each band is then proportional to the square of the signal magnitude in each band.

Designing these analog filters well — so that they precisely align to the desired band centers, roll off at precisely the right places, include all spectrum energy, avoid excessive overlap, and scale gains to correctly reflect the signal power in each band — turns out to be a remarkably difficult design problem. There is little wonder that high-quality spectrum analyzer instruments have always been relatively expensive.

Digital processing technology is a practical alternative today. Digital filtering can approximate the behaviors of the traditional analog filters. This approach also turns out to have its own share of difficulties, however. Analog filter banks have a natural logarithmic character, but digital filters do not. The easy methods for mapping the analog designs to digital designs do not produce good results at the band edges, and considerable effort is necessary to individually tune each filter characteristic. Another difficulty is the time/resolution tradeoff. The sampling rates necessary to capture high frequency information produces excessive data sets at low frequencies, so the design turns in to a multi-rate as well as a multi-band problem.

The approach discussed in this note is based on FFT analysis. This shares the resolution and time scale difficulties of other digital approaches. However, it is appealing because the difficult filter design work is eliminated. An FFT acts like a huge bank of very precisely tuned digital filters. The challenge is to properly identify the desired 1/3 octave bands.

Preprocessing? or Postprocessing?

Performing the octave analysis on a DAP sometimes make good sense; sometimes it does not. It depends on the application.

If the venerable HP analyzer instruments had produced only a stream of raw measurement numbers, not an immediate presentation of the spectrum results, it would have been considered a disastrous failure. Yet, that kind of thing is tolerated as the norm in the present day of "virtual instruments."

You can do some really advanced analysis using the appropriate GUI tools in your PC host environment. There is a price, however, in terms of computing resources, complexity, and... price. Sometimes you just want the spectrum numbers without all of that overhead. Then it can make sense to embed the processing with your data acquisition.

The FFT analysis provides detailed spectrum information. What then remains is the bookkeeping details for distinguishing the frequency bands. The next section provides more information about how this is done.

Octave Analysis via FFT

The new and more general `MIXRFFT` command in the DAPL 3000 system avoids restrictions that would make the 1/3 octave processing more complicated. The slightly increased computational load is fully within the capabilities of xDAP systems.

Some of the features of this new FFT processing:

- *Mode free.*
- *High precision.*
- *More flexible block sizes..*
- *Flexible data types.*
- *Internally optimized.*
- *Longer blocks.*
- *In-place, low memory utilization.*
- *Cache efficiency.*

The processing of the `MIXRFFT` command is based on the classic Singleton mixed-radix FFT [5].

The second challenge is to analyze the frequencies within the nonlinear spacing of the 1/3 octave bands. The `THIRDOCT` command performs this "bookkeeping" analysis on the spectrum results, to produce the desired 1/3 octave data sets.

A preview: the following two-line configuration provides a rigorous octave analysis covering 34 bands from ANSI band 10 (10 Hz nominal) to ANSI band 43 (20 kHz nominal).

```
// Sampling runs at 48000 samples per second
MIXRFFT(65536,FORWARD,VONHANN, samples,POWER, spectrum)
THIRDOCT(32768,34, spectrum, octaves)
```

One 65536-term block is analyzed every 1.36 seconds. As you can see, the application is quite trivial to set up, but the sampling rate and the FFT block size are not arbitrary. In fact, they must satisfy some very specific requirements.

How It Works: Internal Math

A special observation simplifies the "bookkeeping" problem of octave analysis. There is a relationship between frequencies in an FFT spectrum and frequencies in an octave. Consider the set of 12 linearly-spaced frequencies for the octave ranging from 10 Hz to 20 Hz.

```
10.000, 10.833, 11.666, 12.500, 13.333, 14.166,
15.000, 15.833, 16.666, 17.500, 18.333, 19.166, 20.000, ...
```

Compare these to the logarithmic spacing of band center frequencies necessary for 1/3 octave analysis.

```
   10.000, - - - - - - - - 12.599, - - - - - - - -
   - - - - 15.874, - - - - - - - - - - - - - - - - 20.000, ...
```

The correspondence between the 1/3 octave band locations and the 12 uniformly-spaced frequency locations is remarkable. This can be considered a convenient mathematical accident. This grouping of 3 + 4 + 5 frequency samples happens to work almost perfectly to span the first octave. Align the first octave of interest to this group of frequencies, and you have a valid octave analysis for this band. Doubling the frequencies shifts to the next octave, and this octave is covered by 24 frequency samples with grouping of 6 + 8 + 10. And this pattern continues. If the first octave is correctly aligned, the rest will follow. If you have enough terms and the right sampling rate, this pattern can cover the full spectrum you want to analyze. A tabulation given in the appendix lists all of the standard frequency bands your application might want to cover.

The band alignment is not perfect, so there are some predictable biases in the signal power measurements for each band. If you care about this, you can apply a compensation factor to "level the gain" for signal power indicated in each band.

Band terms	Correction factor to apply	dB correction to apply
3	+3.80%	+0.337 dB
4	-1.78%	-0.152 dB
5	-0.99%	-0.086 dB

Configuring FFT Length and Sample Rate

This section explains how to correctly specify the FFT length and the corresponding sampling rate to align to 1/3 octave standard ANSI bands. However, first check the table of pre-determined configurations below. You are likely to find the combination you need, and if so, you can dispense with the details in the rest of this section.

Sampling and Block Size - Configuration Table

ANSI bands	Covered frequency range	Sampling rate to use	Block size	Freq misalign
10 to 43	8.8 to 22.4k	48000 samples/sec	65536	0.02%
10 to 42	8.8 to 17.9k	48000 samples/sec	65536	0.03%
10 to 41	8.8 to 14.2k	30000 samples/sec	40960	0.03%
10 to 40	8.8 to 11.3k	24000 samples/sec	32768	0.03%
11 to 43	11.0 to 22.4k	48000 samples/sec	52000	0.00%
11 to 42	11.0 to 17.9k	48000 samples/sec	52000	0.00%
11 to 41	11.0 to 14.2k	30000 samples/sec	32500	0.00%
11 to 40	11.0 to 11.3k	24000 samples/sec	26000	0.00%
12 to 43	13.9 to 22.4k	48000 samples/sec	41250	0.02%
12 to 42	13.9 to 17.9k	48000 samples/sec	41250	0.02%
12 to 41	13.9 to 14.2k	30000 samples/sec	25740	0.22%
12 to 40	13.9 to 11.3k	24000 samples/sec	20625	0.05%
13 to 43	17.5 to 22.4k	48000 samples/sec	32768	0.02%
13 to 42	17.5 to 17.9k	48000 samples/sec	32768	0.03%
13 to 41	17.5 to 14.2k	30000 samples/sec	20480	0.03%

13 to 40	17.5 to 11.3k	24000 samples/sec	16384	0.03%
14 to 43	22.1 to 22.4k	48000 samples/sec	26000	0.02%
14 to 42	22.1 to 17.9k	48000 samples/sec	26000	0.03%
14 to 41	22.1 to 14.2k	30000 samples/sec	16250	0.00%
14 to 40	22.1 to 11.3k	24000 samples/sec	13000	0.00%
15 to 43	27.8 to 22.4k	48000 samples/sec	20625	0.02%
15 to 42	27.8 to 17.9k	48000 samples/sec	20625	0.03%
15 to 41	27.8 to 14.2k	30000 samples/sec	12870	0.21%
15 to 40	27.8 to 11.3k	24000 samples/sec	10296	0.21%

Find the desired band coverage in the configuration table. Enter the *block size* number as your FFT block length, and configure the `TIME` or `SCAN` command in your DAPL 3000 configuration to sample each data stream at the given sample rate.

General design

If you do not see a configuration you can use in the table above, you can apply the following steps to determine a suitable sampling rate and block size.

1. In the frequency band table, find the band center frequency for the lowest frequency band you want to analyze.

2. Compute the FFT resolution number to align this low band.

```
    resolution =  1.36491 * (9.843 / band center frequency)
```

3. In the frequency band table, find the upper frequency limit of the highest band you want to analyze.

4. Select a suitable sampling frequency to use, based on the classic Nyquist criterion.

```
        sample rate >= 2 * high frequency
```

5. Compute the corresponding FFT block size required.

```
        Nfft = sample rate * resolution
```

6. Adjust the block length to a "nice" number with a factorization compatible with `MIXRFFT`.

There is some flexibility in choosing the sampling rate and FFT block size. For example, you might choose a conventional number and accept a slight compromise in the block size, or you can pick the most efficient block size and operate at whatever sampling rate the analysis suggests. Close approximations will yield an analysis with slightly shifted frequency bands. So, for example, if your calculations say the block size should be 16249 values, using a block size 16250 or 16384 should produce satisfactory results with frequency alignment off by a fraction of a percent.

Here is an example of the calculations, based on the first configuration in the *typical configuration* table above. The low frequency band has a band center frequency 10.0, so the *resolution* figure is computed as follows.

```
    resolution =  1.356491 * 10.0/10.0  =  1.356491
```

For the highest band ANSI 43, the highest frequency is 22627, so by the Nyquist criterion the sampling frequency must be greater than 45254. The commonly used audio sampling rate of 48000 is selected. Given this choice, the block size should be

```
    block size = 48000 * 1.36491  =  65516
```

The block size 65536, which is an exact power of 2, is an excellent approximation and a natural choice.

Configuring the 1/3 Octave Bookkeeping

After performing the FFT power spectrum analysis, all that remains is to accumulate the power terms in accordance with the table of 1/3 octave bands shown in the appendix.

A DAPL custom processing command `THIRDOCT` [4] makes this easy and convenient, but it is not currently included in the DAPL system commands. It is available for free download. The `THIRDOCT` command knows the band grouping of terms as shown in the table, and it will step through an FFT power spectrum to accumulate the 1/3 octave spectrum groups automatically. You need to tell it:

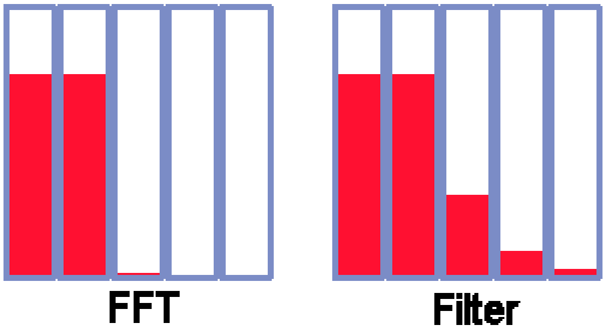
- which bands you want it to compute
- the size of the input power spectrum block, so that it knows how many padding terms to ignore at the end of each FFT block.

Reducing resolution

One thing that you cannot do with the FFT-based 1/3 octave analysis is match the results of typical filter-based instruments. The resolution of the FFT-based analysis is vastly better.

It is not because the instruments are bad. The filters are designed that way. Though the standards [3] do not require this, many implementations mimic the analog filters described as a reference implementation. This preserves a degree of consistency with historical measurement instruments, even if this is not the best result possible.

A well-configured FFT will respond a very small amount to off-center frequencies present in an adjacent FFT location. But two locations away: no significant response. The narrowest 1/3 octave band spans 3 FFT locations, so we can state simply that *there is no relevant interaction beyond one neighboring 1/3 octave band.* For a typical instrument using a 6th order Butterworth bandpass filter, *bands three steps removed* will interact. The following graphic is shown very close to true scale, for a case of frequencies located very close to the band edge.



If you really prefer to match the behavior of a historical instrument, a Data Acquisition Processor could apply appropriate resolution-reduction adjustments as an additional post-processing step. You would have to determine the matrix describing how much each band filter responds to signals that are out-of-band. Applying this matrix would spread the well-isolated results of the FFT analysis and make the results closely match the requisite instrument response. Keep in mind that this is a process of throwing away information. A processing command to do this is not currently implemented.

Conclusions

We have shown how a rigorous 1/3 octave analysis can be configured trivially, after some careful (but easy) preliminary calculations to guarantee alignment with ANSI band location standards. The FFT analysis for the application requires very long data blocks, and to obtain a best tradeoff between block sizes and sampling rates there must be flexibility. The `MIXRFFT` command recently added to the DAPL system has all of this. Preprocessing calculations performed by your xDAP system can eliminate separate steps for transferring and logging the raw measurements that you don't need to preserve. The onboard data reduction makes your host software applications easier – a nice benefit in exchange for the 2 lines of configuration script that you need to add.

- *An xDAP system can support a 10 to 20000 Hz nominal frequency range spectrum analysis on 8 channels simultaneously, with ultra-low distortion and very high band resolution.*

The `THIRDOCT` command is available now for download from the Microstar Web site.

Footnotes and References:

1. *"Octave Analysis Explored,"* Kurt Veggeberg, Evaluation Engineering, August 2008, pp 40-43.
2. The Microstar Laboratories web site presents the article *IEC651A — A Processing Command for A-weight Audio Filtering* at http://www.mstarlabs.com/dsp/iec651a/iec651.html. This provides some background information about A-weighting in general, and discusses a filter implementation available for download from the Microstar Laboratories web site.
3. ANSI s1.11-1986, "Specification for octave-band and fractional-octave-band analog and digital filters"
ANSI s1.11-2004, "Specification for octave-band and fractional-octave-band analog and digital filters"
IEC 1260:1995-07, "Electroacoustics - Octave Band and Fractional Octave Band Filters"
IEC 1260:1995-07 Annex A
4. The downloadable module `THIRDOCTM`, with the `THIRDOCT` command discussed in this article, is available for free download from the Microstar Laboratories web site.
5. The Singleton mixed radix FFT was originally developed in FORTRAN IV by R. C. Singleton at Stanford Research Institute, in 1968. A version was later and published in Programs for Digital Signal Processing, DSP Committee, ed., IEEE Press, 1979. Other variants have been distributed, for example, at Netlib.

APPENDIX - ANSI 1/3 octave band table

This table lists:

1. the 1/3 octave frequency bands by sequence number
2. the nominal center frequency assigned to each band
3. the band center locations for mathematically precise 1/3 octave spacing
4. the band edges bounding each of the mathematical frequency bands
5. the number of FFT locations needed to cover the band
6. the total number of FFT locations

ANSI 1/3 Octave Filter Bands

ANSI band #	nominal center	exact center	exact edges	number of FFT bins	highest FFT bin
10	10.0	9.843	8.769 11.049	3	15
11	12.5	12.401	11.049 13.920	4	19
12	16	15.625	13.920 17.538	5	24
13	20	19.686	17.538 22.098	6	30
14	25	24.803	22.098 27.840	8	38
15	31	31.250	27.840 35.077	10	48
16	40	39.373	35.077 44.194	12	60
17	50	49.606	44.194 55.681	16	76
18	63	62.500	55.681 70.154	20	96
19	80	78.745	70.154 88.388	24	120
20	100	99.213	88.388 111.362	32	152
21	125	125.000	111.362	40	192

			140.308		
22	160	154.490	140.308 176.777	48	240
23	200	198.425	176.777 222.725	64	304
24	250	250.000	222.725 280.616	80	384
25	315	314.980	280.616 353.553	96	480
26	400	396.850	353.553 445.449	128	608
27	500	500.000	445.449 561.123	160	768
28	630	629.961	561.123 707.107	192	960
29	800	793.701	707.107 890.899	256	1216
30	1000	1000.000	890.899 1122.462	320	1536
31	1250	1259.921	1122.462 1414.214	384	1920
32	1600	1587.401	1414.214 1781.797	512	2432
33	2000	2000.000	1781.797 2244.924	640	3072

34	2500	2519.842	2244.924 2828.427	768	3840
35	3150	3174.802	2828.427 3563.595	1024	4862
36	4000	4000.000	3563.595 4489.848	1280	6144
37	5000	5039.684	4489.848 5656.854	1536	7680
38	6300	6349.604	5656.854 7127.190	2048	9728
39	8000	8000.00	7127.190 8979.696	2560	12288
40	10000	10079.368	8979.696 11313.708	3072	15360
41	12500	12699.208	11313.708 14253.379	4096	19456
42	16000	16000.000	14253.379 17959.393	5120	24576
43	20000	20158.737	17959.393 22627.417	6144	30720
44	25000	25398.417	22627.417 28508.759	8192	38912

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